Predicting Gas Apparent Permeability of Shale Samples: A Novel Analytical Approach

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Abstract

Natural gas production of the United States from shale resources increased from 4 percent of total gas production in 2005 to 40 percent in 2012. These resources are different from conventional hydrocarbon resources due to the presence of extremely tight organic pores and low permeabilities. Presence of the nanopores may cause rarefaction effects, especially in laboratory conditions, which increases the effects of temperature and pressure on the apparent permeability of shale samples. In order to determine the permeability of these resources, laboratory measured apparent permeabilities, if conducted in low pressure and temperature, need to be extrapolated to reservoir conditions. In addition, gas flow in low pressures has important applications in predicting the gas production rates from unconventional reservoirs.

Analytical methods for estimating gas apparent permeability (AP) of shale have been already proposed, e.g. Navier-Stokes and Advective -Diffusive Models (ADM); however, they are valid for a limited range of Knudsen numbers (Kn < 0.5) and they have oversimplifying assumptions that overestimate the mass flux (or permeability) of nanopores. In addition, their results do not show the effect of temperature and gas molecular weight on AP.

The presented work aims to develop an analytical model for gas apparent permeability of nanopores which is valid for Knudsen number up to unity. Solutions to the Regularized 13 (R13)-moment equations (extension of Grad’s 13-moments equations) provide a reliable tool to derive an analytical model for gas AP in nanotubes. The novelty of this work is that we provide an analytical model for gas AP which is valid for higher range of Knudsen numbers (by comparing with the kinetic data) in contrast to the previously developed analytical models. The new model is used to predict the impact of controlling parameters such as temperature, pressure, molecular weight, pore size, and Tangential Momentum Accommodation Coefficient (TMAC) on gas AP. It is shown that the gas molecular weight and temperature have significant effect on gas apparent permeability at low pressures. The effect of adsorption on AP of nanotubes is studied by employing the experimental Langmuir isotherms of different shale samples.

The bundle of tubes method is used to compare R13 AP model with the experimental data of a Marcellus shale core plug. The model’s AP results for Nitrogen and Carbon Dioxide agree with the experimental measurements.
**Introduction**

Shale gas reservoirs has become a significant research area due to their unique transport properties. These unconventional resources are characterized by their extremely low intrinsic permeability (Civan et al., 2010; Javadpour, 2009; Loucks et al. 2012) and therefore, the gas flow regime changes from Darcy ($Kn < 0.01$) to slip and transition regimes ($0.01 < Kn < 10$) (Roy et al., 2003) especially in laboratory conditions. In slip and transition flow regimes, the rarefaction effects play significant role in gas transport where the apparent permeability (not intrinsic) is affected by operating parameters such as pore pressure, and temperature (Darabi et al., 2012; Chalmers et al., 2012; Passey et al., 2010). Hence, in order to determine the AP of shale resources, the laboratory measured APs need to be extrapolated to reservoir conditions. Therefore, an accurate representative model is required that accounts for these rarefaction effects in high Knudsen gas flow, i.e. $Kn>0.01$.

This study present a new analytical model (R13 AP) for gas apparent permeability of nanotubes. The Regularized 13 (R13)-moment equations, which was developed by Struchtrup et al. (2003) and linearized by Taheri et al. (2009), are solved to find an analytical model for gas AP of nanotubes. The linearized R13 mass flux results are compared with linearized Boltzmann kinetic data and is shown to be valid for all ranges of Knudsen number up to unity in contrast to the existing analytical models (Javadpour, 2009; Azom et al., 2012; Singh et al., 2014). Gas molecular weight and temperature are shown to have significant effect on gas AP and TMAC effect on gas AP is demonstrated to be minimal for its experimental range (0.8-1) at lower Knudsen numbers. An adsorption model is introduced and its effect on gas AP is studied using several experimentally measured Langmuir isotherms of shale samples. The R13 AP model results is compared with apparent permeability measurements of a Marcellus shale core sample. A bundle of tube method is then used to compare R13 AP model with AP experimental measurements.

Several attempts have been made to develop analytical models for flow in extremely low permeability porous media while accounting for rarefaction effects. Klinkenberg (1941) took into account the slip effect and experimentally found that the gas AP is approximately a linear function of the reciprocal mean pressure. Beskok and Karniadakis (1999) presented a formulation for flow through a capillary tube having a circular cross-section based on the Navier-Stokes equations (unified flow model). Due to non-equilibrium behavior of the gas on length scales of the order of mean free path, the Navier-Stokes equations fail to describe flows in rarefied gases (Taheri et al., 2009). Javadpour (2009) developed an analytical model for gas AP of nanotubes. The total mass flux is determined by superposition of the Knudsen diffusion and advective gas flow driven by pressure forces. Although the gas-wall interaction can be adjusted in advective flow term, the fully diffusive boundary condition is assumed in Knudsen diffusion term (Mason and Malinauskas, 1983). In addition, the model is overestimating the mass flux for Knudsen numbers larger than 0.1. Civan (2010) employed the unified flow model of Beskok and Karniadakis (1999) to find an equation for gas AP. He developed correlations for Klinkenberg slippage factor as well as tortuosity. Azom and Javadpour (2012) modified the Javadpour (2009) model by considering the real gases. Darabi et al. (2012) employed the Javadpour (2009) model and modified it by considering effect of wall roughness on Knudsen diffusion. Singh and Javadpour (2013) considered a linear summation between the advective flow and Knudsen diffusion. The newly developed model (NAP) by Singh et al. (2014) claims to remove the dependency of Javadpour (2009) model from TMAC. However, as mentioned previously, the model contains the assumption of fully diffusive boundary condition in Knudsen diffusion term. Furtheremore, this model overestimate the gas AP for Knudsen numbers above 0.1.

Other authors also proposed numerical and experimental models for gas apparent permeability (Zhu et al., 2009; Tang et al. 2005; Akkutlu and Fathi, 2012; Sakhaee-Pour and Bryant, 2011; Jones et al., 1980; Civan, 2010; Florence et al., 2007; Sampath and Keighin, 1982).
The Boltzmann equation is valid for rarefied gas flows for all ranges of Knudsen number; however, the numerical simulation of the Boltzmann equation is computationally expensive. In order to overcome this problem, the Chapman-Enskog method (Cercignani, 1976; Chapman and Cowling, 1970; Ferziger and Kaper, 1972) and the method of moments of Grad (Grad, 1949, 1958; Muller and Ruggeri, 1998) were developed. Unfortunately, the Chapman-Enskog method, in which the phase density is expanded in powers of the Knudsen number, becomes linearly unstable for the second and third order expansions (Burnett and Super-Burnett) and can not be used for simulation of processes with small wavelengths (Bobylev, 1982; Rosenau, 1989; Struchtrup and Torrilhon, 2003). In the method of moments of Grad (Struchtrup and Torrilhon, 2003), the Boltzmann equation is transferred to a set of partial differential equations for the moments of differential equations (set of moment equations). The authors previously presented an analytical model for gas AP of nanochannels (Kazemi et al., 2015). In this article the Regularized 13 (R13)-moment equations are solved for a nanocapillary with cylindrical geometry. A gas apparent permeability model is then derived and the sensitivity of operating parameters such as pressure, temperature, molecular weight, and temperature on gas AP is studied. The effect of adsorption on gas AP is also addressed. The R13 AP model is then compared with AP measurements of a Marcellus shale core plug.

**Mathematical Model**

Gas flow in nanoscale mediums is associated with non-equilibrium behaviors of flow adjacent to solid surfaces such as near wall temperature and velocity gradients. These non-equilibriums effects form the Knudsen layer which have a thickness of two mean free paths (Ohwada et al., 1989; Taheri and Struchtrup, 2009). The degree of non-equilibrium behavior of gas flow is determined using the Knudsen number (Kn) which by definition is the ratio of gas mean free path to the characteristic length of the medium (for example channel width or tube radius). It is well known that the classical hydrodynamics (Navier-Stokes equations) cannot describe the gas flows with high Knudsen number (Struchtrup, 2005; Cercignani, 1988). The Boltzmann equation has proved to be useful in describing the dilute gas flows beyond the capabilities of the Navier-Stokes equations. It describes the gas dynamics using the particle velocity distribution function,

\[
\frac{\partial f}{\partial \tau} + \xi_k \frac{\partial f}{\partial x_k} + G_k \frac{\partial f}{\partial \xi_k} = J(f,f)
\]  

(1)

where \( f(x_k, t, \xi_k) \) is the particle distribution function, \( \xi_k \) is the microscopic particle velocity, \( G_k \) is the external force, and \( J(f,f) \) describes the change of phase density due to collision between particles. Although the Boltzmann equation is useful for a wide range of Knudsen numbers, its solutions are complex both numerically and analytically. In order to reduce the computational costs, Grad (1949) developed a reduced order method (13-moment) which is valid for \( 10^{-3} < kn < 1 \). The Grad’s 13-moment equations was regularized by Struchtrup and Torrilhon (2003) in order to stabilize the differential equations. The new set of equations for 13-moments exhibit the Knudsen boundary layer and are linearly stable.

In order to derive the moment equations, the Boltzman equation is multiplied by a polynomial of the microscopic velocity, \( \xi \), and integrated over the velocity space (Grad, 1949). The polynomials are particularly selected to be 1, \( \xi \), and \( \xi^2/2 \) to generate the mass, momentum, and energy equations. After multiplying and integrating over the velocity space, thirteen differential equations are derived (Struchtrup, 2005).

In this work, the R13 differential equations are linearized and solved analytically to determine the macroscopic variables (Taheri, 2010). After linearization and non-dimensionalization, the following three differential equations needed to be solved to find an equation for the fluid velocity,
where,

\[
\frac{\partial}{\partial \bar{r}} \left( \frac{1}{\bar{r}} \right) \bar{\sigma}_{rz} = -\bar{\sigma}_i, \tag{2}
\]

\[
-\bar{\sigma}_i + \frac{1}{2} \left( \frac{\partial}{\partial \bar{r}} + \frac{1}{\bar{r}} \right) R_{rz} = - \frac{\pi}{2} \frac{1}{Kn} \bar{q}_z, \tag{3}
\]

\[
\frac{2}{5} \frac{\partial \bar{q}_z}{\partial \bar{r}} + \frac{\partial m_{rz}}{\partial \bar{r}} + \frac{m_{rz} - m_{\phi \phi z}}{\bar{r}} = - \frac{\pi}{2} \frac{1}{Kn} \bar{\sigma}_{rz} - \frac{\partial \bar{v}_z}{\partial \bar{r}} \tag{4}
\]

where,

\[
R_{rz} = - \frac{14}{5} \frac{2}{\pi} Kn \frac{\partial \bar{q}_z}{\partial \bar{r}}, \quad m_{rz} = \frac{8}{5} \frac{2}{\pi} Kn \left( \bar{G}_i + \frac{5}{4} \bar{\sigma}_{rz} \right), \quad m_{\phi \phi z} = - \frac{2}{5} \frac{2}{\pi} Kn \left( \bar{G}_i + \frac{5}{4} \bar{\sigma}_{rz} \right) \tag{5}
\]

The reference equilibrium state which is used for non-dimensionalization and linearization of the R13 equations is defined by \( \rho_0, \theta_0, v_0 \). The variables with hat sign in Eq. 2, 3, and 4 are dimensionless variables and are defined as follows,

\[
\bar{v}_i = \frac{v_i}{\sqrt{\theta_0}}, \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\rho_0 \theta_0}, \quad \bar{q}_i = \frac{q_i}{\rho_0 \theta_0^3}, \quad \bar{G}_i = \frac{\bar{G}_i}{\theta_0} \tag{6}
\]

where \( \bar{\phi} \) is the characteristic length of the tube. To simplify the notation, the hat signs indicating the dimensionless variables are omitted in the reminder of the paper. For a two-dimensional tube (Fig. 2), solving equations 2, 3, and 4 for velocity and heat flux, the following solutions are derived,
It should be mentioned that the tangential heat flux, $q_z$, in isothermal gas flow, which vanishes in classical hydrodynamics, is predicted in the R13 method. Since the solution must be finite at $r = 0$, the constants $C_1$ and $C_3$ must be zero. The other two constants ($C_2$, $C_4$) should be determined from the boundary conditions. The Maxwell accommodation model is employed in which a fraction gas particles are specularly reflected from the wall and the rest of them attain the thermal equilibrium with the wall (diffusively reflected). In the later one, the particles reflect according to the Maxwell distribution function at the local surface temperature,

$$v_z = C_4 + \frac{G_z}{4K_n} r^2 - \frac{C_1}{K_n} \ln(r) - \frac{2}{5} q_z.$$

(9)

It should be mentioned that the tangential heat flux, $q_z$, in isothermal gas flow, which vanishes in classical hydrodynamics, is predicted in the R13 method. Since the solution must be finite at $r = 0$, the constants $C_1$ and $C_3$ must be zero. The other two constants ($C_2$, $C_4$) should be determined from the boundary conditions. The Maxwell accommodation model is employed in which a fraction gas particles are specularly reflected from the wall and the rest of them attain the thermal equilibrium with the wall (diffusively reflected). In the later one, the particles reflect according to the Maxwell distribution function at the local surface temperature,

$$f_w(\xi) = \frac{\rho_w}{m(2\pi \theta_w)^{\frac{3}{2}}} \exp\left(-\frac{(\xi - v_w)^2}{2\theta_w}\right),$$

(10)

$$f(\xi) = \chi f_w(\xi) + (1 - \chi) f_{gas}^*(\xi).$$

(11)

Where $m$ is mass of the gas, $\rho_w$ is the gas density at the wall vicinity, $\chi$ is the Tangential Momentum Accommodation Coefficient (TMAC) that is the fraction of diffusively reflected molecules, $\theta_w$ is the wall temperature in energy unit, $f_{gas}^*(\xi)$ and $f_w$ are the velocity distribution functions of specularly and diffusively reflected particles, respectively. Multiplying the boundary condition with microscopic velocity polynomials and integrating over the velocity space, the integration constants, $C_2$ and $C_4$, can be determined.

The reduced mass flux consequently can be calculated by integrating the velocity equation over the tube radius,
The apparent permeability can be determined using the Darcy’s equation as,

$$G^* = 2\sqrt{2} \int_0^r rv_z \, dr.$$  \hspace{1cm} (12)

The apparent permeability can be determined using the Darcy’s equation as,

$$k_{app} = \frac{G^* \mu r}{p_m} \sqrt{\frac{RT}{2M}}$$ \hspace{1cm} (13)

Dimensonless permeability is defined as the the ratio of apparent permeability to its absolute value,

$$k_d = \frac{k_{app}}{k_\infty}$$ \hspace{1cm} (14)

where the absolute permeability for a tube is $$k_\infty = \frac{r^2}{8}$$. Using equation 13, the apparent permeability can be derived as,

$$\frac{k_{app}}{k_\infty} = 1 + 4.41 \, Kn + 1.35 Kn^2$$ \hspace{1cm} (15)

with $$Kn = \sqrt{\frac{\pi RT \mu}{2M p_m r}}$$. For the remainder of the paper, we use the above equation for apparent permeability calculations.

**Results and Discussions**

In this section, the derived analytical model for apparent gas permeability of shale is validated with linearized Boltzmann data and experimental mass flux measurements through microtubes. Furthermore, the effect of channel size, pressure, gas molecular weight, TMAC, temperature, and adsorption on apparent gas permeability is addressed. The mathematical model results are compared with the AP experimental measurements of Marcellus shale core sample at the end of this section.

**Model Validation**

The mathematical model is compared with the kinetic data of linearized Boltzmann along with existing models for apparent permeability of shale in Figure 1. The R13 model reduced mass flux agrees well with linearized Boltzmann data of Cercignani and Daneri (1963) for Knudsen numbers up to unity. The R13 AP model is also compared with the experimental data obtained by Perrier et al. (2011) for gaseous transition flow in microtubes. These experiments were performed in isothermal conditions at 296.5 K with Argon as the gas in a microtube with 50 $\mu$m diameter. Figure 1 suggests that the R13 AP model have a very good agreement with experimental data of Perrier et al. (2011).

**Gas Apparent Permeability**

In this section, the effect of controlling parameters, namely, temperature, molecular weight, pressure, and TMAC, on gas apparent permeability are investigated.

**Temperature effect**

The dimensionless apparent permeability of Argon is plotted against the temperature for tube diameters 5, 10, and 100 nm at 1 MPa in Figure 2. The R13 AP model shows that the effect of temperature is significant on apparent permeability of shale gas reservoirs especially for smaller tube sizes. The R13 AP model shows a more significant impact of temperature in comparison to other existing models. For a tube with 5 nm diameter, the R13 AP model demonstrate a 25% increase in apparent permeability for a temperature change of 300 to 400 °K. Therefore, temperature impacts should be considered in order to extrapolate the measured apparent permeability in laboratory to reservoir condition.

**Molecular Weight effect**

The apparent permeability measurements in laboratory are usually performed with different gas types such as Helium, Argon, or Nitrogen. Although the absolute permeability, $k_\infty$, would be the same for these gases,
the measured apparent permeability at low pressures are different from each other. Figure 3 shows the dimensionless apparent permeability plotted against the gas molecular weight at 2 MPa and 350 °K for tube diameters 5, 10, and 100 nm. R13 AP model shows that by increasing the gas molecular weight from 16 to 40 gr/mol, the apparent permeability reduces by 90% and 60% for tube diameters 5 and 10 nm, respectively. Therefore, shale samples with lighter gases produce gas at higher flow rates for the same pressure gradient.

![Figure 3—Dimensionless permeability vs. molecular weight for different tube diameters at P=2 Mpa and 350 °K](image)

**Pore pressure effect**
Dimensionless permeability of Argon is plotted against the inverse of pressure for tube diameters 10, 20, 50, and 100 nm at 350 K. The apparent permeability does not change significantly with pressure for tube diameters 50 and 100 nm. However, the impact of pressure on apparent permeability is significant for tube diameters 10 and 20 nm. The increase in the dimensionless permeability is more pronounced for tubes with smaller diameter, which is in agreement with existing models and experiments (Javadpour, 2009; Singh et al., 2014; Fathi et al., 2012). Also, as the pressure increases, the apparent permeability approaches the absolute permeability, i.e. $\frac{k_{app}}{k_\infty} \rightarrow 1$.
**TMAC effect**

Tangential Momentum Accommodation Coefficient (TMAC) should be determined in order to determine the boundary conditions properly. TMAC depends on several factors such as gas type, pressure, and surface properties (Agrawal and Prabhu, 2008). Experimental measurements show that the TMAC values ranges from 0.8-1 (Sreekanth, 1969; Ewart et al., 2007a; Maurer et al., 2003; Agrawal and Prabhu, 2008; Ewart et al., 2007b). The effect of TMAC on apparent permeability is plotted in Figure 5 for Knudsen numbers 0.1, 0.5, and 1. As it is shown, the TMAC effect on apparent permeability is not significant for Knudsen numbers less than 0.5. The TMAC reduces by 60% for Kn=1; however, this reduction is 30% for Kn=0.1.

![Figure 4](image1.jpg)

**Figure 4**—Effect of pore pressure on apparent permeability.

![Figure 5](image2.jpg)

**Figure 5**—Effect of TMAC on gas apparent permeability.
**Adsorption effect**

When a molecule hits the pore surface, loses its energy, and stays on the surface for a reasonable amount of time (few nanoseconds), the molecule “sticks” to the solid wall (Masel, 1996). The sticking is usually due to physisorption or chemisorption processes. The models accounting for adsorption describe the sticking by isotherms that relate the amount of adsorbate taken up by adsorbent. The most well-known adsorption isotherm is the Langmuir adsorption isotherm (Attard and Barnes, 1998; Auroux, 2013; Gregg and Sing, 1983; Langmuir, 1918). This isotherm accounts for monolayer adsorption which takes place at the solid surface consisting a distribution of non-interacting sites. The Langmuir hyperbolic adsorption isotherm is,

\[
\alpha = \frac{Kp}{1 + Kp}
\]

where \(K\) is the equilibrium constant and \(\alpha\) is the surface coverage and represents the surface fraction occupied by adsorbed molecules. Effect of adsorbed layer on channel apparent permeability are investigated by comparing apparent gas permeability values of a channel without an adsorbed layer. The effect of adsorbed layer on reducing the tube cross section area is demonstrated in Figure 6. It should be mentioned that the interaction of gas molecules with adsorbed molecules is ignored in this study. However, a diffusive reflection is expected when the free gas molecules hit the adsorbed molecules. This is in agreement with the original definition of diffusive reflection by Maxwell in which he assumes that a stratum of gas molecules are presented on the surface and the free gas molecules hit this surface, after which all direction of its velocity become equally probable (Maxwell, 1879). The apparent permeability of nanochannel, which is a function of surface coverage, is calculated as,

\[
k_{app} = \frac{1}{\frac{\alpha(p)}{k_{ads}} + \frac{1 - \alpha(p)}{k_f}}
\]

where \(k_{ads}\) is the apparent permeability of section of tube with adsorbed gas and \(k_f\) is the apparent permeability of non-adsorbed tube.

The thickness of the adsorbed layer is assumed to be 0.7 nm considering the carbon dioxide molecule size (0.35 nm) (Kurniawan et al., 2006). Both the surface coverage (\(\alpha\)) and apparent permeability are functions of pressure. For investigating the impact of adsorbate layer on apparent permeability, the isotherm measurements performed by Heller and Zoback (2014) on Barnett, Eagle Ford, Marcellus, and Montney shale samples using CO\(_2\) (Figure 7) are used.
The ratio of apparent permeability of the tube to the apparent permeability of non-adsorbed tube is plotted against the adsorption surface coverage for two tube diameters 3 and 5 nm in Figure 8. The ratio decreases as the surface coverage increases for both tube diameters non-linearly. The effect is more significant for smaller tube diameter, i.e. D=3nm. The apparent permeability reduces by 26 and 41% for tube diameters 5 and 3 nm, respectively. The apparent permeability reduction for surface coverages up to 50% is less than 15%. The apparent permeability curves of Barnett and Montney reservoirs are overlapping since their Langmuir adsorption isotherms are similar to each other.

Model Comparison with Core Plug Experiments
The R13 AP model is compared with Marcellus shale core plug experiments performed by Zamirian et al. (2014). The experiments performed under the steady state condition on a Marcellus shale core sample...
with low total organic content. Helium (He), Nitrogen (N₂), and Carbon Dioxide (CO₂) were used to measure gas apparent permeability for a pore pressure range of 100 to 500 psia (0.7 to 3.5 MPa) at 70°F. The pore size distribution of sample (Figure 9) was measured using low temperature Nitrogen adsorption technique and the sample porosity was determined to be 4.8%.

![Figure 9—Pore size distribution of Marcellus shale sample (Zamirian et al., 2014)](image)

In this study, the R13 AP model is compared with measured apparent permeability of CO₂ and N₂. Because the Helium experiments were performed at Knudsen numbers more than 1.5 and our model is valid for Knudsen numbers up to unity, we were not able to compare R13 AP model with Helium experiments.

Due to complexity of the porous media morphology, the flow is usually conceptualized using the bundle of tubes model. In this method, the pore spaces are modeled as capillary tubes. Simplicity is the most important advantage of the method while it suffers from several shortcomings such as disregarding the morphology of the porous media, tortuosity, and considering a unidirectional permeability (Sochi, 2009).

In the case of a tube existing in a solid block, the absolute permeability of the block is determined as (Kozeny, 1927; Johnson et al., 1986),

\[ k = k_{oo} \frac{\phi}{\tau} \]

where \( k_{oo} = \frac{r^2}{8} \), \( \phi \) is the porosity, and \( \tau \) is the tortuosity of the solid block structure. In order to determine the permeability of a core sample with a known pore size distribution, the bundle of tube method can be employed as (Figure 10),

\[ k_{app} = \frac{\phi}{\tau} \sum_{i=1}^{N} k_{oo} \lambda_i \]

where \( \lambda_i \) is the fraction of each pore with specific radius determined from pore size distribution.
The $\phi$ ratio is called the obstruction factor (Civan, 2011) in which a very small value can be expected for rock with low porosity and high tortuosity. A good agreement was found with an obstruction factor of 0.027 which results in a tortuosity of 1.75 by fitting the R13 AP model with Nitrogen experimental data (Figure 11). The analytical model matches the experimental data for pressures down to 1 MPa, below which the Knudsen number goes beyond the validity limit of the model.

Since adsorption was observed in shale permeability measurements using CO$_2$, the adsorption model proposed above is employed. The Langmuir isotherms that was found by Heller and Zoback (2014) for Marcellus shale is used in this study because both of the Heller and Zoback (2014) and Zamirian et al. (2014) samples have low TOC. Using the same tortuosity ($\tau=1.75$) value used for matching R13 AP model with Nitrogen experimental data, the apparent permeability values predicted by R13 AP model matches the CO$_2$ adsorption experimental data. The CO$_2$ apparent permeability model without considering the adsorption is also plotted in Figure 11 (green dashed line), for which apparent permeability values fall within 1.03 to 1.1 greater than the values with adsorption. A particularly noteworthy feature of R13 AP model is that both the N$_2$ and CO$_2$ models (without adsorption) extrapolate to the same absolute
permeability value of 40 nD. The absolute permeability of CO₂ model with adsorption is determined to be 38 nD which is less than its value for nonadsorbing case. Zamirian et al. (2014) estimated the absolute permeability for CO₂ to be 124 nD using the double-slippage method (Fathi et al., 2012), which is almost three times larger than the R13 AP predicted values. Unlike the double slippage model, R13 AP model honors the values of apparent permeability at high pressure and estimates lower absolute permeability values.

Conclusions

An analytical model for gas apparent permeability of nanotubes is developed. The R13 mass flux results matches the linearized Boltzmann data for Knudsen numbers up to unity. The results show that the existing models overestimate the mass flux through nanotubes and are valid for a limited range of Knudsen numbers. The R13 AP model shows that the impact of molecular weight and temperature on gas apparent permeability of nanotubes is significant and can have substantial impact on gas production rate predictions of shale gas reservoirs. The apparent permeability increases as the temperature increases or the gas molecular weight decreases. The model also demonstrates that the effect of Tangential Momentum Accommodation Coefficient on gas apparent permeability is significant for flows with Knudsen number larger than 0.5.

The proposed adsorption model show that the effect of adsorption on gas apparent permeability is significant for adsorption surface coverages more than 50%. In addition, the apparent permeability changes non-linearly with adsorption surface coverage.

The R13 AP model agrees well with apparent permeability measurements on a Marcellus shale core plug using N₂ and CO₂ gases. The extrapolated absolute permeability values for both Nitrogen and Carbon Dioxide are the same assuming nonadsorbing gases. The absolute permeability of Carbon Dioxide with adsorption is determined to be slightly lower. Comparing with double slippage model, R13 AP model honors the values of apparent permeability at high pressure and estimates lower absolute permeability values.

Nomenclature

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\( v_k \) Macroscopic velocity in direction \( k \)
\( v_w \) Wall velocity

\textbf{Greek Symbols}

\( \alpha \) Surface coverage
\( \chi \) Tangential Momentum Accommodation Coefficient (TMAC)
\( \Delta P \) Pressure difference across the channel
\( \lambda_i \) Percentage of a pore with channel size \( L_i \)
\( \phi \) Porosity
\( \rho_0 \) Reference density in equilibrium state
\( \rho_w \) Gas density at wall
\( \sigma_{ij} \) Stress tensor
\( \tau \) Tortuosity
\( \theta \) Temperature in energy unit
\( \theta_w \) Wall temperature in energy units
\( \xi_k \) Microscopic particle velocity in \( k \) direction

\textbf{References}


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