Effects of image resolution and numerical resolution on computed permeability of consolidated packing using LB and FEM pore-scale simulations

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A R T I C L E   I N F O

Article history:
Received 19 January 2013
Received in revised form 21 May 2013
Accepted 27 May 2013
Available online 6 June 2013

Keywords:
Lattice Boltzmann method
Finite-element method
Image-based simulations
Numerical resolution

A B S T R A C T

Image-based pore-scale modeling has become an important tool for studying fluid transport and other phenomena in porous media. Spatial resolution of the digital images used for modeling is critical not only because it dictates the scale of features that can be resolved, but also because for most techniques there is at least some relationship between voxel size in the image data and numerical resolution applied to the computational simulations. In this work we investigate this relationship using a computer-generated consolidated porous medium, which was digitized at voxel resolutions in the range 2–10 μm. These images, which are free of experimental and segmentation errors, are then used to compute permeability and tortuosity using lattice Boltzmann (LB) and finite elements methods (FEM). Results show how changes in computed permeability are affected by image resolution (which dictates how well the pore geometry is approximated) versus grid or mesh resolution (which changes numerical accuracy). For LB, the image and grid resolution are usually taken to be the same; we show at least one case where effects of grid and image resolution appear to counteract one another, giving the mistaken appearance of resolution-independent results. For FEM, meshing can provide certain attributes (such as better conformance to surfaces), but it also adds an extra step for error or approximation to be introduced in the workflow. Results show that performing grid coarsening on the FEM mesh caused a reduction in computed permeability, but in this particular case the effect is related to tightening of the pore space rather than loss of numerical accuracy.

Published by Elsevier Ltd.

1. Introduction

In geoscience and petroleum engineering studies, estimation of effective rock properties can often be performed using more than one approach, each with various benefits and drawbacks. Experimental approaches are preferred if it is not possible to account for all relevant physics by an equation or model; however, they tend to be time consuming and expensive. Empirical approaches provide simplicity, but are usually valid for limited conditions and sacrifice a connection to the underlying physics. For certain properties such as permeability of the porous media, numerical simulation using digital images has become a credible alternative, enabled by improvements of 3D imaging techniques, numerical methods, and computing power. Appealing aspects of this approach include the ability to probe pore-scale physics at a level not possible with traditional experiments and the ability to perform an endless set of numerical tests without degrading or altering the sample. Considerations that can limit this digital approach include whether the imaging technique can resolve all relevant characteristic scales in the pore space and whether numerical algorithms are able to accurately model the physical processes.

For fluid flow problems, two main approaches to image-based pore-scale modeling have evolved, which differ based on how the pore space is discretized and how the flow equations are solved. Network models divide the pore space into a collection of pores connected by pore throats [10,9,8]. Hence, the discretization is defined by the pore structure (usually not uniquely). Flow is solved by enforcing mass conservation at the pores and applying Poiseuille flow approximations in the pore throats. The second approach is more traditional computational modeling. The domain is discretized into nodes, voxels, or volume elements, and the resulting grid is used to numerically approximate the relevant partial differential equations for flow.

The latter approach, which we will refer to as a computational fluid dynamics (CFD) approach can be further broken down into techniques that employ regular versus unstructured grids. While this distinction is often worth making in CFD studies, it is
particularly important for image-based modeling because the voxel data from X-ray tomography or similar methods can be used directly as the numerical grid. This gridding approach has become widely used in porous media studies in conjunction with the lattice Boltzmann (LB) method, which has proved to be highly effective for simulating fluid flow through porous media [1,2].

LB has been applied to many porous media flow simulations. Succi et al. [3] were one of the first groups that employed LB to study flow of fluids in complex geometries such as porous media. In their study, they used LB to calculate permeability of porous media. The main result of their work was the LB adherence of Darcy’s equation in complex 3D geometries. Ferreol and Rothman [2] simulated single-phase flow through Fontainebleau Sandstone and found approximately the same permeability values as equivalent finite-difference calculations and laboratory measurements. Jin et al. [4] also presented an integrated procedure for the estimation of the absolute permeability of unconsolidated and consolidated reservoir rock.

Challenges for applying LB to real problems include finite-size effects and relaxation time dependence of no-flow boundaries. In image-based simulations, the accuracy of the calculated macroscopic properties is dependent on the spatial resolution of the rock image [5,6]. However, there is always a trade-off between the image resolution (and the related lattice spacing) versus computational power. Furthermore, in all digital samples, there is a resolution threshold, below which certain flow characteristics, such as recirculation, are not resolved [7].

The alternative to a regular numerical grid is an unstructured mesh used with a compatible computational method such as the finite element method (FEM) or finite volume method (FVM). In the past, difficulties associated with unstructured mesh generation have limited their application to pore-scale modeling problems. However, recent improvements to meshing algorithms and the availability of commercial image-based meshing packages such as ANSYS ICEM CFD have made this less of a concern. In principle, unstructured meshes are well suited for pore-scale modeling for two reasons: an ability to conform to complicated geometries and the ability to employ local spatial refinement. The latter point is particularly significant for microscopically heterogeneous porous media because refinement allows small pores and/or regions critical to transport to be meshed at higher resolutions without introducing additional computational expense where it is not needed. A related advantage is that (depending on how much control the user has over mesh resolution) numerical resolution can be selected based on computational considerations independently of what voxel resolution was used in the digital imaging process.

In this article we study image resolution and numerical resolution to understand the interplay between these two effects and their impact on accuracy. Simulations are performed on a computer-generated sphere pack rather than a real micromtomography image, which allows image resolution to be varied arbitrarily and without any segmentation error. The pack was originally an unconsolidated packing of non-overlapping spheres, but for these simulations the sphere radii were increased to allow overlaps. This step creates a structure that mimics a consolidated material, which is of interest for oil and gas applications and represents a more challenging simulation than in non-overlapping sphere packings because of the smaller and less connected pore space. Permeability and tortuosity computations are made using both LB simulations (structured lattices) and FEM simulations (unstructured tetrahedral meshes).

For clarification, terminology used in the remainder of the paper is the following. The term image resolution is used to denote the size of the voxels used to digitize the image. Poor image resolution implies larger voxels (i.e., a less clear picture of the pore structure); good quality image resolution means smaller voxels. The term numerical resolution is used to denote the characteristic spacing of the numerical discretization. For LB, this spacing is usually a fixed value (the spacing between each lattice node). For unstructured meshes it varies. In this paper we present unstructured mesh resolution by reporting the total number of elements. Since the domain size is fixed, this is an unambiguous quantity.

2. Background

2.1. Image-based flow simulation

A significant amount of past research has been performed to study and validate LB modeling of fluid flow in packed beds. Permeability studies in random sphere packs include [11–20]. Additionally, Pan et al. [21,22], and Stewart et al. [23] studied the effect of sphere size, spatial discretization, and fluid viscosity (relaxation parameter) on the computed permeability of random-sphere packs and Maier et al. [24] investigated flow of single-phase fluid through a column of glass beads.

Similarly, traditional CFD methods have been used to simulate flow in packed beds [25–29]. However, these techniques have not been as widely applied to imaged-based modeling (i.e., where the flow domain is obtained from microCT or similar 3D image data) as LB or network modeling [30].

2.2. Lattice Boltzmann method

LB is a simplification of the Boltzmann equation that solves for particle distribution functions in a discrete phase space. In this method, positions of particles are limited to nodes of a lattice with
equal spacing [31]. Velocity magnitudes in each direction have particular values so that the distance between a particle and its neighbors is equal to the discrete velocity times the time steps.

Lattice Boltzmann models are denoted by \( DdQq \) where \( d \) shows the dimension of the simulation (either 2 or 3) and \( q \) shows the number of discrete directions particle can move. LB consists of two steps: streaming and collision. In streaming, directional specific densities \( f_i \) move towards their nearest neighbors. In the collision step, momentum exchange between the particles takes place due to collisions with each other at a particular node.

The LB equation with streaming and single velocity relaxation operator (LBGK) collision is

\[
f_i(x + \mathbf{e}_i \Delta t, t + \Delta t) = f_i(x, t) - \frac{(f_i(x, t) - f_{\text{eq}}^i(x, t))}{\tau}
\]

where \( \mathbf{e}_i \) are directions in which fluid particles can move and \( f_i \) are the specific distribution functions in velocity space. In this formula, \( f_{\text{eq}}^i \) are the equilibrium distribution functions,

\[
f_{\text{eq}}^i(x) = w_i \left[ 1 + \frac{(\mathbf{e}_i \cdot \mathbf{u})}{c_s^2} + \frac{\mathbf{e}_i \cdot \mathbf{u}}{2c_s^2} \right] \left( \frac{x}{\tau} \right)
\]

where \( w_i \) are weight factors specific to different directions, \( c_s = 1/\sqrt{3} \) is the sound speed in the fluid, \( \mathbf{u} \) is the velocity of the fluid, and \( \rho \) is the density of the fluid. Hydrodynamics variables, density and velocity, are calculated from first and second moments of particle distribution functions.

\[
\rho(x) = \sum_i f_i(x)
\]

\[
\mathbf{u}(x) = \frac{1}{\rho(x)} \sum_i f_i(x) \mathbf{e}_i
\]

Traditional Navier–Stokes equations can be derived from LB equation for weakly compressible fluids in the low Mach number regime [32,5]. Kinematic viscosity can be defined as \( \nu = \frac{1}{3} (\tau - \frac{1}{2}) \) in which \( \tau \) should always be greater than 0.5. Fluid pressure is defined by

\[
p(x) = c_s^2 (\rho(x) - \bar{\rho})
\]

where \( \bar{\rho} \) is the average density of the fluid.

### 2.2.1. Boundary conditions

Due to simplicity of implementing no-flow boundaries using bounce-back schemes for rock/pore interfaces of the porous media images, this scheme was used. In this scheme, distribution functions streamed from a fluid node to a neighboring solid node are bounced back along the same link.

The numerical accuracy of LB is proved to be second-order in space except in the bounce-back boundary regions where the accuracy is only first-order accurate [33,34,37]. There are several more complex no-slip boundary conditions with second-order accuracies. Many of these boundary conditions are only applicable for regular geometries [33,34].

In a bounce-back scheme, the no-flow boundary is assumed to be at a halfway distance between the fluid and solid nodes for simple geometries. However, in more complex geometries such as porous media, the bounce-back scheme puts the no-flow boundary somewhere between the wall and adjacent fluid nodes. The exact position of the no-flow boundary is dependent on the relaxation parameter and the geometry of the system [38,5,39,32]. Image resolution affects this issue because position dependence of the no-flow boundaries are more significant for lower resolution images.

Overall, for flow through porous media studies, the bounce-back scheme is more practical since it simplifies handling of complex pore-matrix boundaries and is computationally efficient. The low-order accuracy improves a few lattice spacings away from the wall and can be compatible with second-order accuracy of LB [32,5,37,40].

In presented simulations, the body force approach, which is an imitation of pressure boundary conditions at the inlet and outlet, is used in LB. In this approach, a pressure gradient acting on the fluid is replaced by an external force [41,2]. At the inlet and outlet, extra layers of 10 lattice spacings were added and periodic boundary condition was applied. The body force was not applied to the added layers.

### 2.2.2. Finite-size effects

The accuracy of the LB method depends on how well the numerical domain is resolved by lattice spacings. In cases that the numerical domain is not well-resolved, finite-size effects or Knudsen flow behavior may mask the true results [5,6].

To avoid problems, the characteristic length of the numerical domain must be at least a few orders higher than the mean free path in order for results to describe hydrodynamic behavior of the flow. Relaxation time also impacts finite-size effects. It controls the mean free path [5].

Finite-size effects generally decrease as smaller relaxation times are used [42]. Holdych et al. [43] used a recursive application of the LB equation to achieve explicit forms for the effective variable stencils in LB schemes and obtained modified equations for fluid flow models. They found that truncation errors are second-order in space and are proportional to low-order polynomial orders of the relaxation time. They also assigned different values of the relaxation times for different flow regimes to minimize truncation errors.

Finite-size effects are particularly important in the simulation of flow through porous media where the numerical domain consists of many irregular pore bodies with converging–diverging geometries. In order to determine whether the finite-size effects are large, fine graining can be performed, meaning each voxel in the image contains multiple lattice sites for the numerical simulation. However, when this approach is used on a regular grid the computational expense can be significant.

### 2.2.3. Parallelization study and scaling performance

In this study, the PALABOS (Parallel Lattice Boltzmann Solver) code, which is a parallelized LB code for solving flow problems and is available from the web address www.lbmethod.org/palabos, has been used. All flow simulations were carried out on Louisiana Optical Network Initiative (LONI) resources.

Strong and weak scaling performance of the PALABOS code is tested for a 3D rectangular channel flow and shown in Fig. 1. In these tests, input/output operations of each processor are minimized.

The strong scaling test is to determine how well the code performs with a problem of fixed size while increasing the number of cores. Fig. 1a shows the wall clock time spent by processors versus the number of processors used for the computation in a log–log plot. In an ideal strong scaling test, the wall clock time is considered to diminish linearly as number of processors increase, giving a log–log plot with a slope of −1. The slope calculated for the code has a slope of −1.05.

In weak scaling, the work load assigned to each processor should remain constant. In other words, each processor has the same amount of work to do, such that no slowing should be observed for the ideal case and a log–log plot of computational time versus number of processors should have a slope of zero. In Fig. 1b, log–log plot showed a slope of 0.04, which exhibits weak scalability. Based on these performance tests, the PALABOS code exhibits good scaling performance.
or Taylor–Hood ele-
Galerkin approach is employed that uses a P2
CFD applications[44], its application to pore-scale flow simulations
applied in a more general way to image-based pore-scale modeling.
However, these approaches have not been
relatively simple 2D geometries, or simple configurations of spheri-
tially ties mesh resolution to the number of voxels. Although no
leads to an extremely high number of surface elements, and essen-
tially ties mesh resolution to the number of voxels. However, in image-based modeling the mesh is created from the voxel image rather than the original material, so this argument is not as compelling as it would be otherwise.
A voxel-based meshing approach described by Young et al. [47] has been built on a marching-cubes-type approximation of voxels on the void-solid interface, with a look-up table that splits those voxels intersected by a surface into tetrahedral elements. Away from the interface the approach has the ability to transition to hexahedral elements. The authors noted that their strategy resulted in an overestimate of surface area which can prejudice physics based simulations that depend on surface area. It was also mentioned that increasing the voxel to diameter ratio does not necessarily lead to an improvement in the approximation of surface area, which is consistent with the work of Lindblad [48]. In their voxel-based meshing approach the lack of adaptivity also leads to an extremely high number of surface elements, and essentially ties mesh resolution to the number of voxels. Although no physics was included in the study, element quality was assessed.

We have used two approaches to generating unstructured tetra-
hedral meshes from segmented image data. One approach is an in-
house algorithm, which maps the porous medium surface onto an
experiment image resolution, segmentation, and the inherent
approximation when arbitrary surfaces are described using a voxel
format (regardless of the resolution). When meshes are created
from digital images, geometric and topologic parameters are al-
ttered further. An unstructured mesh typically has a different pore
volume and surface area than the digital image that is used as the
template. Connectivity can also change, especially in cases of cor-
nor-to-corner voxel connections, which even for the highest-quality
images do not provide a definitive answer as to whether a
physical connection existed in the original porous media. One
can make an argument that an unstructured mesh is a better tool
for characterizing the pore space than voxel data. For instance, sur-
faces of rounded grains or crystalline surfaces (that are not aligned
with the principle voxel dimensions) can be captured by surface triangulations more effectively than by regular voxels. However, in image-based modeling the mesh is created from the voxel image rather than the original material, so this argument is not as compelling as it would be otherwise.

2.3.1. Meshing
The aforementioned reluctance to use FEM for image-based
pore-scale modeling is largely related to the added challenge posed
by mesh generation in a heterogeneous, interconnected flow do-
main. Part of the challenge is algorithmic: implementation of mesh
generation for these structures can be difficult. However, this part
of the problem is becoming less burdensome because of modern
off-the-shelf meshing routines that operate directly on voxel
images. The more difficult aspect of the problem is understanding
and quantifying whether the mesh structure effectively captures
the pore structure.

For image-based modeling, the digital image used as the basis
for the structural model is already an approximation to the actual
pore structure, which encompasses errors associated with the
experiment image resolution, segmentation, and the inherent
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We have used two approaches to generating unstructured tetra-
hedral meshes from segmented image data. One approach is an in-
house algorithm, which maps the porous medium surface onto an
existing unstructured mesh using a distance function. The second
is a Computer Aided Design (CAD) based approach, which we have run using the commercially available Avizo software. Generally, our in-house algorithm gives more control over both local refine-
ment and constraining specific parameters such as surface area and
porosity. However, the CAD-based approach performs better in terms of retaining connectivity present in the voxel image and
conforming to the voxel boundaries. In this paper all results are based on the latter CAD-based approach, implemented through
Avizo. The meshes are transferred to an in-house data format prior
to running the Stokes flow solver. This step is for convenience in

2.3. Finite-element for pore-scale flow
While the finite element method is widely used for traditional
CFD applications [44], its application to pore-scale flow simulations
has been very limited. This is unfortunate because one of the main
attributes of FEM is its ability to operate on unstructured meshes,
which in turn allows local refinement to help address the wide var-
iation in pore size and local velocity in a microscopically heteroge-
eous porous media. Some notable studies have considered
relatively simple 2D geometries, or simple configurations of spheri-
cal particles in 3D [45,46]. However, these approaches have not been
applied in a more general way to image-based pore-scale modeling.

In this paper we solve the Stokes equations by FEM. A Bubnov-
Galerkin approach is employed that uses a P2,P1, or Taylor–Hood ele-
ment. This choice of element implies that a linear approximation is
assumed for pressure and a quadratic approximation is assumed for
velocity. Both pressure and velocity are defined at the vertices of
the tetrahedral element, while the additional degrees of freedom
required for the quadratic velocity approximation are added at the
mid-points of each tetrahedral edge. No slip conditions are im-
posed on all interior surfaces and on the four no-flow boundaries.
Constant traction (normal component) is imposed at the inflow
and outflow locations. See Section 2.3.2 for more discussion of
boundary conditions. Details of the numerical implementation
can be found in Lane [44].

Fig. 1. Strong and weak scaling study of PALABOS code.
applying boundary conditions for our flow solver; the change in data format does not alter the mesh structure.

2.3.2. Boundary condition

As noted in a recent investigation [49], the choice of appropriate boundary conditions for modeling pore scale flow is somewhat arbitrary. This can be attributed to a lack of detailed information at geometrically complicated inlet and outlet planes required to specify either the velocity profile or pressure distribution as well as on the other four side walls.

As with the inlet and outlet, the side walls are generally arbitrary cuts through the pore space, and the boundary conditions for the hypothetical flow patterns into and out of these side walls (i.e., had the porous medium not been cut) cannot be known. Hence, the generally accepted approach is to consider the side walls of the flow domain as no-slip surfaces.

The no-slip boundary condition simplifies the problem, the tradeoff being confinement of the fluid and increased drag force acting on the fluid. Ideally, this is mitigated by considering large enough samples so that results become independent of boundary conditions. An argument can be made, in fact, that the no-slip condition on the side walls is the same boundary condition routinely applied in laboratory core flood experiments in which the confining sleeve is a no-flow and no-slip boundary condition along the external surface of the core. (Albeit, the physical dimensions of a laboratory core are typically an order of magnitude larger than the comparable tomography image.)

The alternative to the no-slip approach is to impose periodic boundary conditions. However, for real materials this requires mirroring or an artificial buffer region and either of these approaches has its own drawbacks.

For the inlet and outlet faces, the most common objective is to apply upstream and downstream pressures. This presents some difficulty because for FEM the direct boundary condition that can be applied is total stress. For inlet/outlet pores with high viscous stresses the pressure and the normal component of total stress can differ significantly. One way to address this problem is to add non-porous flow sections upstream and downstream of the domain. This approach minimizes viscous stresses in the inlet and outlet (because of the large dimensions and slow flow relative to the pore dimensions) and thus allows a traction boundary condition to be applied as a substitute for the desired hydrostatic pressure condition. An approach similar to this has been used by [49,50]. In studies of flow through sphere packs the extension of the computational domain prevented particles from intersecting the inlet and outlet planes which simplified both the implementation of boundary conditions and the identification of boundary elements.

We have tested this same strategy previously and found no significant difference in predicted permeability based on the addition of inlet and outlet sections. Hence, our standard approach to pore-scale FEM modeling is to apply boundary conditions directly to the pore space on the inlet and outlet faces of the domain: total normal traction is set equal to the negative of the desired applied pressure. This approach avoids the added computational expense of modeling the inlet and outlet flow sections.

2.4. Permeability calculation

Permeability of a sample of porous media is defined as

\[ K = -\mu \frac{\langle u \rangle}{\nabla P} \]  

(6)

where \( K \) is the permeability tensor of the porous medium, \( \langle u \rangle \) is the average velocity of fluid, \( \mu \) is the viscosity of the fluid, and \( \nabla P \) is the dynamic pressure gradient in the fluid. In this work, permeability is computed in the same manner as in a laboratory experiment: an applied pressure drop is imposed and flowrate is measured (or vice versa). These two values, the fluid viscosity, and the sample dimensions are then used to compute permeability.

2.5. Tortuosity calculation

Tortuosity quantifies the degree that fluid must flow on a tortuous or indirect path as it passes along a linear distance of a porous medium. One definition of tortuosity is the square of the actual distance fluid travels in the domain to the length of the domain in the direction of pressure gradient:

\[ T = \left( \frac{L_x}{L} \right)^2 \]  

(7)

where \( L_x \) and \( L \) are the length of the domain and the traveled distance by the fluids, respectively [51]. Nabovati and Sousa [52] presented a volume-averaged estimate of tortuosity from numerical pore-scale simulation results. They suggested

\[ T = \frac{\sum_{i,j,k} u_{avg}(i,j,k)}{\sum_{i,j,k} |u(i,j,k)|} \]  

(8)

where \( x \) is the leading flow direction.

3. Samples

Computer-generated structures provide a number of advantages for testing pore-scale modeling algorithms. The most obvious advantage is the ability to fully control the pore structure. Another advantage related to image-based modeling is that geometric-based data (e.g., locations and sizes of spheres in a random packing) can be converted to voxel data at any desired image resolution without segmentation error.

Computer-generated sphere packings have been widely used to simulate granular materials. In some cases, unconsolidated sphere packs have been modified using procedures that mimic daigenetic processes, thus producing consolidated materials [53–55]. The simplest modification is to increase the sphere radii to allow sphere-sphere overlaps, which decreases porosity. In the current study, this approach was applied to a random sphere pack with 1000 particles. The original packing had a cube-shaped domain size 1 mm on each side. The sphere size distribution had a mean diameter of 100 \( \mu m \) and a standard deviation equal to 25% of the mean. The pack was fully periodic and had a porosity of 36.5%. The diameters of the spheres were then increased until the porosity was reduced to 14.5%.

To allow studies of image resolution, this artificially consolidated structure was converted into five voxel images, with voxel resolution varying from 2 \( \mu m \) (an image consisting of 500\(^3 \) voxels), to 10 \( \mu m \) (100\(^3 \) voxels). Fig. 2a and b show a cross-section of the poorest image resolution and best image resolution, respectively. At the best image resolution, the solid/void surfaces are smoother, while the voxelization clearly compromises the pore structure when the resolution is poor. Porosity and specific surface areas of these images are tabulated in Table 1. Porosity variation is small, which reflects the fact that even at low image resolution the portions of solid voxels residing in the pore space is somewhat offset by the portions of void voxels residing inside the spheres. Specific surface areas are calculated based on the algorithm presented in Thompson [56], and this value increases with increasing resolution but begins to converge at higher resolutions.

Fig. 3a and b are histograms showing pore and pore-throat size distributions in the final structure. These values were obtained by creating a physically-representative pore network from the 2-\( \mu m \)-voxel data set. The resulting network is described by a variety of rigorous geometric measurements made in the pore space.
These measurements include the size of maximal inscribed spheres in each pore and maximal inscribed spheres at the tightest point in connecting pore throats. Hence, this type of network data is ideal for characterization of the pore space.

Overall, the pore-throat size distribution, porosity, permeability, and image resolutions for this artificial system are reasonably consistent with values that would be expected for image-based modeling of moderate- to high-permeability sandstones (i.e., order hundreds of millidarcy). This higher permeability rock, in turn, is the most amenable to quantitative prediction of properties using digital imaging.

3.1. LB results

3.1.1. Image resolution

LB simulations were performed on the five data sets. Recall that the overall domain dimensions remain constant (a cube 1 mm on each side), but voxel dimensions range from 10 µm at the coarsest to 2 µm at the finest resolution. In a typical LB approach the numerical lattice is coincident with the voxel grid. Hence, numerical resolution improves (i.e., grid spacing decreases) as image resolution improves.

Z-direction fluid flow is simulated for the five images. Flow is body-force driven and the lateral boundaries are no-flow boundaries. Periodic boundary conditions are applied at the inlet and outlet of the simulated samples. (The physical domains are

<table>
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<th>L/a</th>
<th>a (µm)</th>
<th>ϕ (%)</th>
<th>S_v (1/m)</th>
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<td>100</td>
<td>10</td>
<td>14.33</td>
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<tr>
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<td>14.34</td>
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<td>300</td>
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<td>14.34</td>
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<tr>
<td>500</td>
<td>2</td>
<td>14.34</td>
<td>20705.89</td>
</tr>
</tbody>
</table>

Fig. 2. Images of the random sphere pack with two different resolutions. (a) and (b) show the images with resolutions a_0 and a_0/5, respectively. Due to the higher resolution in (b), roughness of the staircase-like representation is more smoothed out compared to the low-resolution image.

Fig. 3. Pore and pore-throat size distributions for the computer-generated porous medium. Values are the diameters of maximal inscribed spheres in the pores (a) and at the tightest constrictions in the connecting pore throats (b). Measurements were made using a network-generation algorithm on the 2 µm digital image.
periodic structures.) Calculated $z$-direction permeabilities of these images are plotted in Fig. 4.

In general, permeability decreases as the image resolution and grid spacing improves (the two resolution effects are inseparable in the current set of simulations). The data are also broken down by relaxation time. Calculated permeability is more sensitive to relaxation time in the lower-resolution images: at $10 \mu m$ resolution, permeability varies by 62%; at $2 \mu m$, the variation is only 14%. These variations are due to both relaxation time dependence of finite-size effects and the no-flow boundary condition (bounce-back scheme) used in these simulations. In images with larger lattice spacings (lower resolutions), pore spaces are characterized by fewer grids, which makes both the position-dependence of no-flow boundaries and the finite-size effects more significant.

For the particular case of relaxation time equal to 1.1, the computed permeability is essentially constant over the range of image/lattice resolutions. However, despite these bulk permeability values being consistent, further investigation showed that the pore-scale flow fields are significantly different from one another. This behavior is discussed further in the next section. The computed permeability of the consolidated sphere pack at the highest image and numerical resolution is found to be 0.375D ($\pm 0.025D$) and referred to as the true permeability in the following sections.

### 3.1.2. Grid refinement

In the above analysis (and many LB studies), it is not possible to separate the effects of improved image resolution and improved numerical resolution because the numerical grid is coincident with the voxel grid. Hence, to quantify this effect for the current data, we fix the image resolution and increase the numerical resolution by placing multiple lattice points within each voxel.

We use the lowest resolution image ($100^3$ voxels), and then perform simulations with 2, 3, 4, and 5 lattice nodes per linear voxel dimension. This makes the five different grid resolutions correspond to the same grid resolutions shown in Fig. 4, but the pore structure does not change. In the subsequent reporting, we refer to these five simulations as $1X$, $2X$, $3X$, $4X$, and $5X$, respectively.

Comparing the results of Figs. 4 and 5 shows that computed permeability is more sensitive to grid spacing alone than the combined effects of image resolution plus grid spacing that was discussed in the previous section. Specifically, for the fixed pore structure represented by the low-resolution image, computed permeability decreases by a factor of between 2 and 3 depending on the relaxation time. Further, even at the smallest lattice spacing ($2-\mu m$ in the $5X$ image), the results do not yet appear to have converged numerically. The effect is more pronounced for the smaller relaxation times.

To put this result in a more general context, consider the low-resolution versus high-resolution voxel images as two different porous media (rather than two different approximations to the same structure). In this hypothetical case, the voxel data are considered to be exact representations of two different pore structures defined by the low- and high-resolution images. Accordingly, one would expect them to have different permeabilities. Comparing the most reliable $a = 2$ results from Fig. 4 versus Fig. 5 indicates that the true permeability of the high-resolution structure is approximately twice the permeability of the low-resolution structure. This finding gives insight into the relatively flat $\tau = 1.1$ curve in Fig. 4 that was discussed earlier. Specifically, as grid spacing decreases, the changing numerical resolution corresponds to lower computed permeabilities. However, as grid spacing decreases the physical flow structure also changes, and in this particular case the physical change in the voxel structure leads to increased permeability. The two effects appear to essentially offset one another for the $\tau = 1.1$ case.

These observations are significant because permeability is often used as a surrogate measure of confidence/quality in image based modeling. (For instance, validation is often based on permeability agreement with an experimental value, consistency in permeability for two different image resolutions, or permeability independence versus numerical resolution). The $\tau = 1.1$ result in Fig. 4 is a case that would misleadingly suggest convergence in the computed permeability values.

Tortuosity and $z$-direction permeabilities are tabulated in Table 2. Tortuosities are less sensitive to changes of relaxation times in comparison with permeabilities. Tortuosity also remains relatively constant for different image resolutions. However, as numerical resolution is changed in the $100^3$ image, tortuosity varies between 1.57 and 1.69, which indicates a more tortuous path that fluids take to flow through the low-image-resolution structure. Fig. 6a and b show velocity streamline plots at the same grid spacing, but for the high- and low-image resolutions respectively. As can be seen in these figures, poor image resolution can lead to lost connections as well as a coarser representation of surface.

![Fig. 4. Calculated $z$-direction permeabilities for different lattice spacings and different relaxation times. For improved resolution (smaller lattice spacing), permeability decreases and permeability becomes less sensitive to relaxation time. For each resolution, calculated permeabilities are higher for lower relaxation times. For the relaxation time of 1.1, calculated permeabilities for all the image resolutions are similar, although microscale flow behavior is not consistent.](image1)

![Fig. 5. Calculated permeabilities of the lowest resolution image at five different numerical resolutions (1X, 2X, 3X, 4X, and 5X images) versus lattice spacing size and different relaxation times. For all relaxation times, calculated permeability decreases with the grid spacing. Results are more sensitive to relaxation time value at coarse lattice spacing.](image2)
structure, both of which contribute to the differences in permeability when comparing equal grid spacings but different image resolution.

Fig. 7a and b depict normalized z-direction velocity \( \left( v_z/v_y \right) \) in the same location in the xy plane, but for the high-resolution versus 5X simulation respectively. Velocities are significantly higher than the average because this region connects two pores via a narrow pore throat. Plots of velocity profiles in these two images for different relaxation times are shown in Fig. 7c and d. The effect of the relaxation time on the velocity profiles is not significant.

3.2. FEM results

The image- versus numerical-resolution issues are quite different when using FEM for pore-scale modeling because of the mesh generation step. On one hand, the FEM (or other mesh-based methods) has the disadvantage of having the additional meshing step prior to simulation. This makes the mesh two steps removed from the original porous media structure, meaning it will be affected by any resolution, imaging, or segmentation errors contained in the voxel data, but may have additional geometric or topologic errors associated with mapping the voxel structure onto the unstructured mesh. On the other hand, the ability to use an unstructured mesh allows image resolution and numerical resolution to be decoupled more effectively than with lattice based techniques. Other advantages include the ability to employ local mesh refinement and the ability to create smooth surfaces, even in cases where a fixed-resolution, non-smooth voxel image is the template.

3.2.1. Image resolution

Adjusting parameters in the mesh generation algorithms allows mesh resolution (elements per volume) to be controlled more-or-less independently of the underlying image resolution. This attribute implies that improving image resolution does not force the size of the computational problem to increase. This idea is illustrated for the current problem in Fig. 8, which shows the same section of an image at two resolutions. The poor resolution case is from the 10 \( \mu m \) data set. The high resolution case is from the 2 \( \mu m \) data set. The meshes overlaying each voxel image contain similar sized elements, which qualitatively highlight how element size can be decoupled from the voxel size.

Fig. 9a shows computed permeability as a function of number of elements in the mesh. Results are shown for all five voxel resolutions. Meshes with the largest number of elements tend to conform to the voxel-based pore structure most accurately and will also minimize numerical error. For the best image resolution and the largest number of elements, the permeability agrees well with the 2-\( \mu m \) resolution results from the LB simulations. Also, as the image resolution decreases but numerical accuracy remains high, permeability decreases by a factor of approximately two, which is consistent with the LB simulations.

For fixed image resolution and decreasing number of elements, permeability also decreases. Notably, this trend is opposite what was observed in the LB simulations for the low image-resolution data set. Further insight into the FEM behavior is provided by Fig. 9b, which is a plot of effective porosity versus element resolution. (Effective porosity is the mesh volume divided by the total volume. This mesh-based porosity generally differs somewhat from the voxel-based porosity because the unstructured mesh is a different approximation of the void structure compared to the voxel pore space.) For the meshing algorithm used in this study, coarsening of the mesh causes a simultaneous decrease in mesh volume and effective porosity. Hence, the decrease in permeability with decreasing number of elements is due at least in part to the fact that flow is being modeled in a tighter pore space.

Finally, we note that the change in permeability could also be tied to numerical accuracy in addition to the porosity effect. A series

<table>
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<tr>
<th>Lattice spacing (( \mu m ))</th>
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Fig. 6. Velocity streamline plots from the highest image resolution resolution data set (a) and the 5X sample (b). Numerical grid spacing in the two cases are the same, but image resolution results in a significant difference in resolved pore structure.
of tests were conducted in which the surface triangulation (on the interior grain surfaces) was not allowed to change, while the number of elements in the interior mesh was varied by more than an order of magnitude. This test allows mesh resolution to vary while porosity and the meshed surface structure to remain fixed. In contrast to the Fig. 9a results, the fixed-porosity test showed little change in permeability as a function of number of elements over the range tested. This result is also consistent with tests in a variety of ducts, which shows that modeling viscous flow using the P2P1 element results in excellent accuracy even for few elements across the duct width. These latter results can be found in Lane [44].

4. Discussion and conclusions

A computer-generated packing of overlapping spheres was used as a surrogate for a consolidated granular material and digitized to create voxel images at five different resolutions. The resulting set of images was used to independently probe the effects of both image resolution and numerical resolution (grid spacing or mesh size) on permeability for LB and FEM flow simulations. This approach is important because in practice it is often difficult to separate the effects of these two factors.

LB and FEM results converge to similar permeability values of approximately 0.35 Darcy when both image resolution and numerical resolution are highest. As image resolution becomes coarser, the pore structure and topology are compromised by the larger voxels. Evidence from both simulation methods shows that the lower-quality images have a correspondingly lower physical permeability (i.e., the actual permeability of the compromised structure). However, the situation is more complex when image resolution and numerical resolution both come into play (as is usually the case in practice).

When the LB lattice was taken to be the same as the voxel grid, the computed permeability decreased as image/lattice resolution improved. Additionally, the coarser-resolution images made the simulation results more sensitive to the choice of relaxation parameter.

For the choice of relaxation parameter $\tau = 1.1$, permeability is relatively insensitive to image/lattice resolution. However, we show that this insensitivity is an artifact: a combined effect of (1) the true permeability of the voxel structure increases with
improved image resolution (because of geometric and/or topologic changes associated with the approximated pore structure) and (2) the computed permeability decreases with improved image resolution (because of better numerical accuracy). The fact that these two effects can offset one another is important to consider because permeability is often used as a surrogate measure to confirm the quality of results in image-based modeling.

FEM simulations require a separate meshing step, which allows numerical resolution to be more independent of the image resolution than with typical LB simulations. However, the numerical and image resolution cannot be separated completely because the mesh is usually generated from a voxel-based digital image, and poor resolution in the image data thus impacts the ability of the mesh to capture the pore structure, even if a large number of elements is used.

In the results shown here, when a large number of elements was used, permeability decreased as image resolution became worse. This behavior reflects structural changes in the voxelized pore space as image quality decreases. For the highest quality images (2 μm resolution), computed permeability decreased as the mesh was made coarser. It appears that this effect is associated with a constriction of the pore space as the mesh is coarsened more so than loss of numerical accuracy. If the surface structure of the mesh is able to accurately capture the pore structure, then the \( P_2 P_1 \) element is able to give excellent results even with a relatively coarse mesh.

Because these results are from computer-generated porous media, they do not contain experimental or segmentation errors that are present in real images. This fact, combined with the realistic values for permeability and resolution that were chosen, means that the results presented here provide good benchmarks for the expected performance of LB and FEM results in image-based modeling applications.

Based on these concluding remarks, following recommendations can be made:

- Due attention must be paid to the quality of the images that are provided for running pore-scale simulations. The quality of these images is one of the most important factors affecting the quality of the simulation results.
- Ideally, if the imaging tools have the capability of taking images of the porous media at two or three different resolutions, it might be a good practice to generate images with different

![Fig. 8](image1.png) ![Fig. 9](image2.png)
resolutions and check the simulation results dependence on the image quality. However, for the images of real rocks, image segmentation step might complicate the process further.

• From the simulation results on the same images, both numerical methods (LB and FEM) seem to predict similar values of permeability whenever the image resolution is high (which also implies a rather good numerical resolution).

Acknowledgments

The authors thank Michal Kulczewski for his contribution to the preliminary version of this work, which was presented at the International Parallel PDE Conference in 2012. The authors are grateful for the financial support of the Louisiana Optical Network Initiative (LONI) for providing the high performance computing resources in this study. The authors also thank LSU Center for Computation and Technology (CCT) and Louisiana Optical Network Initiative (LONI) for providing the high performance computing resources.

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